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SOUND ATTENUATION IN TUBES DUE TO VISCO-THERMAL EFFECTS

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The propagation of periodic axial sound waves in gases contained in circular cylindrical structures is a function of four parameters: $s = R\sqrt{\rho \cdot \omega/\mu}$, the shear wave number or Stokes number, $k = \omega \cdot R/c$, known as the reduced frequency, $\sigma = \sqrt{\mu \cdot C_p/\lambda}$, the square root of the Prandtl number and $\gamma = C_p/C_v$, the ratio of specific heats. The complete Kirchhoff solution of the sound propagation in tubes problem obtained in 1868 was expressed in terms of these parameters by Tijdeman [1]. In previous works [1, 2] the complex propagation constant was obtained by solving this expression. The results were presented for a limited range in reference [1] and for a broader range in reference [2] but in both cases only for a single fluid, air. In this work the results of a computer code to solve for this propagation constant are presented. The code was used to find the propagation constants (attenuation and phase-shift coefficients) in the range $5 < s < 5000$, $0{\cdot}01 < k < 6$, $0{\cdot}8 < \dot{\sigma} < 1{\cdot}1$ and $1{\cdot}0 < \gamma < 1{\cdot}7$. This range of conditions covers most conditions of interest. The data was then used to fit an equation to express the attenuation and phase-shift coefficients in terms of simpler polynomial-type expressions as a function of these four parameters. A set of tables to obtain the values of the attenuation and phase shift coefficients for values of these four non-dimensional parameters in the above range is also presented. Sound attenuation measurements using superheated R134a refrigerant agrees reasonably well with the computed attenuation in the plane wave region.

1. INTRODUCTION

Sound attenuation of small amplitude acoustic oscillations in tubes with circular cross-section is a classical problem of acoustics. Kirchhoff solved this problem in 1868 [3]. Kirchhoff was able to derive the complete solution to the problem from basic equations but the attenuation coefficient was imbedded in a very complicated transcendental equation.

For almost a century after Kirchhoff's derivation only analytical approximations of his complete solution, in conjunction with other solutions obtained when simplifying assumptions were introduced in the original fundamental equations, were used to estimate the propagation constants.

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Shields *et al.* [4] were the first to numerically solve the original Kirchhoff formulation. Tijdeman [1] determined that the attenuation inside tubes depended on four non-dimensional parameters: $s = R\sqrt{\rho \cdot \omega/\mu}$, the shear wave number or Stokes number, $k = \omega \cdot R/c$, known as the reduced frequency, $\sigma = \sqrt{\mu \cdot C_p/\lambda}$, the square root of the Prandtl number and $\gamma = C_p/C_v$, the ratio of specific heats. (A list of symbols is included in Appendix B). Two of these parameters (σ and γ) can often be considered to remain constant for a given fluid. He then rewrote the transcendental equation in a form which incorporated these non-dimensional parameters and solved the resulting equation for a limited range of conditions for air. Page and Mee [2] extended Tijdeman's work to cover conditions more representative of practical applications by solving Kirchhoff's solution by the Newton–Raphson method as presented in reference [1] to cover all possible conditions of interest. They used polynomials to fit the results and present them in a simple way. The limitation of this work is that the solutions presented are only for air.

Work related to noise in expansion devices and plate evaporators [5–8] currently underway at the Air Conditioning and Refrigeration Center (ACRC) at the University of Illinois required a special experimental set-up to perform acoustic measurements in refrigerant (R134a). This work required the solution of the transcendental Kirchhoff propagation equation for superheated R134a refrigerant inside a tube so that a suitable test section could be designed to make accurate measurements of expansion device generated noise.

A very simple program implemented in Mathematica v 3·0[†] was created to solve Kirchhoff's transcendental equation for the propagation constants (attenuation and phase shift coefficients) given any values of the four relevant parameters k, s, σ, and γ[‡]. A set of tables that cover the range $5 < s < 5000$, $0{\cdot}01 < k < 6$, $0{\cdot}8 < \sigma < 1{\cdot}1$, and $1{\cdot}0 < \gamma < 1{\cdot}7$ is also included in Appendix A. This covers a very wide range of conditions and gases. Using a larger version of these tables, simple polynomial-type expressions were fitted to allow easy determination of the values of the propagation constants given k, s, σ, and γ.

2. ATTENUATION MECHANISMS AND FORMULATION OF THE PROBLEM

The two most important mechanisms of sound attenuation in circular cylindrical tubes are due to the effects of viscosity and heat conduction. The effects of viscosity and conductivity on sound propagation in an open medium, (for example sound waves in air), are much less significant than inside a tube due to the boundary conditions imposed by the tube [9]. There are other mechanisms that contribute to the attenuation of sound in tubes, for example turbulence and convective effects. These mechanisms are present when the fluid inside the pipe is moving (i.e., when

[†] Mathematica Software Package version 3, Wolfram Research, Inc., 100 Trade Center Drive, Champaign, IL 61820, U.S.A. http://www.wolfram.com

[‡] For copies of this program contact the main author or download it from the Journal of Sound and Vibration web page. (http://www.academicpress.com/jsv)

there is a pressure gradient), but these effects seem to be less important than the visco-thermal effects especially when flow velocities are relatively low.

The equations necessary to describe the complete fluid mechanics effects and interactions between the most important parameters of interest (velocity, pressure, temperature, viscosity, and density) are the Navier–Stokes equations, the continuity equation, the equation of state, and the energy equation. Kirchhoff solved this coupled set of equations by introducing the assumption of sinusoidially fluctuating variables. A very detailed description of this solution is presented in reference [3]. The solution of the set of equations is in the form of a complex transcendental equation. This equation was rearranged in reference [1] and is shown below:

$$\begin{aligned} F(s, k, \sigma, \gamma) = \mathrm{i}Z\left(Z - \mathrm{i}\frac{s^2}{k^2}\right)^{-1/2}\left(\frac{1}{\chi_1} - \frac{1}{\chi_2}\right)\frac{J_1(a1)}{J_0(a1)} \\ + \left(\frac{\gamma \cdot k^2}{\sigma^2 s^2} - \mathrm{i}\frac{1}{\chi_1}\right)(Z - \chi_1)^{1/2}\frac{J_1(a2)}{J_0(a2)} \\ + \left(\frac{\gamma \cdot k^2}{\sigma^2 s^2} - \mathrm{i}\frac{1}{\chi_2}\right)(Z - \chi_2)^{1/2}\frac{J_1(a3)}{J_0(a3)} = 0, \end{aligned} \tag{1}$$

where

$$a1 = k\left(Z - \mathrm{i}\frac{s^2}{k^2}\right)^{1/2}, \quad a2 = k(Z - \chi_1)^{1/2}, \quad a3 = k(Z - \chi_2)^{1/2}, \quad Z = \Gamma^2, \tag{2}$$

and χ_1, χ_2 are, respectively, the small and large roots of

$$1 + \chi\left\{1 + \mathrm{i}\frac{k^2}{s^2}\left(\frac{4}{3} + \frac{\gamma}{\sigma^2}\right)\right\} + \frac{\gamma \cdot k^2}{\sigma^2 s^2}\left(\frac{1}{\gamma} + \mathrm{i}\frac{4k^2}{3s^2}\right)\chi^2 = 0. \tag{3}$$

The assumptions utilized in this solution are (1) homogeneous (continuum) medium, (2) small-amplitude sinusoidal disturbances, (3) no reflections (infinitely long tube), (4) axisymmetric disturbances, (5) no steady flow, and (6) no temperature gradient in the fluid. Additional boundary conditions at the rigid tube walls include zero radial and axial velocity. At the tube axis, the boundary condition is zero radial velocity due to axisymmetry. The tube wall conductivity is also assumed large in comparison with fluid heat conductivity. Z is related to the propagation constant Γ as shown in equations (2). Γ is related to the acoustic pressure as shown in equation (4). The acoustic pressure is

$$p_{ac} = (A \cdot \mathrm{e}^{\Gamma\xi} + B \cdot \mathrm{e}^{-\Gamma\xi})\mathrm{e}^{\mathrm{i}\omega t}, \tag{4}$$

where $\xi = \omega \cdot x/c$, and A and B are functions of radius that cancel out when comparing sound attenuation in a tube at different points.

The solution of the Kirchhoff formulation for the attenuation coefficient is valid only when plane waves are being propagated inside the tube. This condition is met only for values of the reduced frequency k less than 1·841. There are many practical situations in which it is necessary to know the propagation characteristics of higher order modes in tubes. For this reason, Page and Mee [2] solved equation (1) for values of k up to six. They stated that results obtained for values of the reduced frequency greater than 1·841 would apply only to axisymmetric disturbances and as such would provide a lower bound limit for the attenuation of the longitudinal disturbance in the tube [2]. Bruneau *et al.* [9] show a formulation useful to determine higher order modes attenuation coefficients. Their results show that most higher order modes attenuate more rapidly than plane waves.

3. NUMERICAL ANALYSIS

Equation (1) was solved following the scheme presented in reference [1]. Basically, this consists of using the iterative Newton–Raphson procedure outlined by

$$Z_{n+1} = Z_n - \frac{F\langle Z_n\rangle}{F'\langle Z_n\rangle}. \tag{5}$$

In equation (5), $F\langle Z_n\rangle$ is equivalent to equation (1), and $F'\langle Z_n\rangle$, the derivative of equation (1) with respect to Z is

$$\begin{aligned}\frac{\partial\cdot F}{\partial\cdot Z} = F'\langle Z\rangle = \mathrm{i}\left(Z - \mathrm{i}\frac{s^2}{k^2}\right)^{-1}\left(\frac{1}{\chi_1} - \frac{1}{\chi_2}\right)&\\ \times\left[\frac{J_1(a1)}{J_0(a1)}\left(Z - \mathrm{i}\frac{s^2}{k^2}\right)^{1/2} - \frac{1}{2}Z\left(Z - \mathrm{i}\frac{s^2}{k^2}\right)^{-1/2}\frac{J_1(a1)}{J_0(a1)}\right.&\\ \left. + \frac{1}{2}kZ\left(1 - \frac{J_1(a1)}{a1\cdot J_0(a1)} + \left(\frac{J_1(a1)}{J_0(a1)}\right)^2\right)\right]&\\ + \frac{1}{2}\left(\frac{\gamma\cdot k^2}{\sigma^2 s^2} - \mathrm{i}\frac{1}{\chi_1}\right)\left[(Z-\chi_1)^{-1/2}\frac{J_1(a2)}{J_0(a2)}\right.&\\ \left. + k\left(1 - \frac{J_1(a2)}{a2\cdot J_0(a2)} + \left(\frac{J_1(a2)}{J_0(a2)}\right)^2\right)\right]&\\ + \frac{1}{2}\left(\frac{\gamma\cdot k^2}{\sigma^2 s^2} - \mathrm{i}\frac{1}{\chi_2}\right)\left[(Z-\chi_2)^{-1/2}\frac{J_1(a3)}{J_0(a3)}\right.&\\ \left. + k\left(1 - \frac{J_1(a3)}{a2\cdot J_0(a3)} + \left(\frac{J_1(a3)}{J_0(a3)}\right)^2\right)\right].&\end{aligned} \tag{6}$$

The Newton–Raphson procedure needs an initial value of Z to start the iterative process. This initial value of Z is provided in the code by using an approximate solution to the problem. The approximate solution used is known as the low reduced frequency solution was obtained by Zwikker and Kosten in 1949 upon the introduction of some simplifying assumptions to the fundamental equations [1]. The equations necessary for this initial value of Z are

$$\Gamma = \sqrt{\frac{J_0(\mathrm{i}^{3/2}s)}{J_2(\mathrm{i}^{3/2}s)}}\sqrt{\frac{\gamma}{n}}, \qquad n = \left[1 + \frac{\gamma-1}{\gamma}\frac{J_2(\mathrm{i}^{3/2}\sigma s)}{J_0(\mathrm{i}^{3/2}\sigma s)}\right], \qquad Z_{initial} = \Gamma^2. \tag{7}$$

4. NUMERICAL RESULTS

The computer code was developed using the Mathematica version 3·0 software package. A copy of the code can be obtained from the authors or on the world wide web (see footnote 7). A very complete set of tables presenting solutions for the attenuation coefficients Γ' and phase shift coefficient Γ'' (real and imaginary parts of the complex propagation constant Γ, respectively) are also presented in Appendix A for different values of k, s, σ, and γ. The first column of the tables shows values for air which agree with previously published results [2]. Following the approach presented in reference [2], a polynomial-type equation was fitted to the data in an effort to simplify the procedure. The objective is to find an expression that represents the propagation constants Γ' and Γ'' as a function of s, k, σ, γ. The best expressions found so far are

$$\Gamma' = 0{\cdot}2436601\left(\frac{\gamma}{s\sigma}\right) + 0{\cdot}8282861\left(\frac{k\gamma}{s}\right)^2 - 0{\cdot}77198\left(\frac{k\gamma}{s}\right)^4$$
$$+ 0{\cdot}4669814\left(\frac{\gamma}{s}\right) + 0{\cdot}07406207\left(\frac{\gamma}{s}\right)^2 + 5{\cdot}932751\left(\frac{\gamma}{s}\right)^3 - 14{\cdot}598\left(\frac{\gamma}{s}\right)^4, \tag{9}$$

$$\Gamma' = 0{\cdot}9999991 + 0{\cdot}1998062\left(\frac{\gamma}{s\sigma}\right) - 0{\cdot}074551\left(\frac{k\gamma}{s}\right)^2 - 0{\cdot}95698\left(\frac{k\gamma}{s}\right)^4$$
$$+ 0{\cdot}5094442\left(\frac{\gamma}{s}\right) + 0{\cdot}1713677\left(\frac{\gamma}{s}\right)^2 + 2{\cdot}5842\left(\frac{\gamma}{s}\right)^3 + 5{\cdot}376701\left(\frac{\gamma}{s}\right)^4. \tag{10}$$

Equation (9) gives an average error of 2·3% over the 6464 data points used to fit it. The matrix of data used to fit the equation is an extended version to the tables presented in Appendix A. The full set of tables is not presented due to space limitations.† Equation (9) performs quite well as Figure 1 shows. Figure 1 shows comparisons of the exact results obtained using equation (1) for air ($\gamma = 1{\cdot}4$, $\sigma = 0{\cdot}842$) to those obtained with equation (9). It should be pointed out that air data was not used to fit equation (9). When equation (9) was used with air for the points in Table 1 (presented in Appendix A), the maximum error was found at $s = 35$, $k = 6$ and was 2·4% and the average error is only 1·2%. Using an approximation by Kirchhoff (wide tube approximation) presented in Table 1 of

†For copies of the excel file with the full set of data contact the main author or download it from the Journal of Sound and Vibration web page http://www.academicpress.com/jsv.

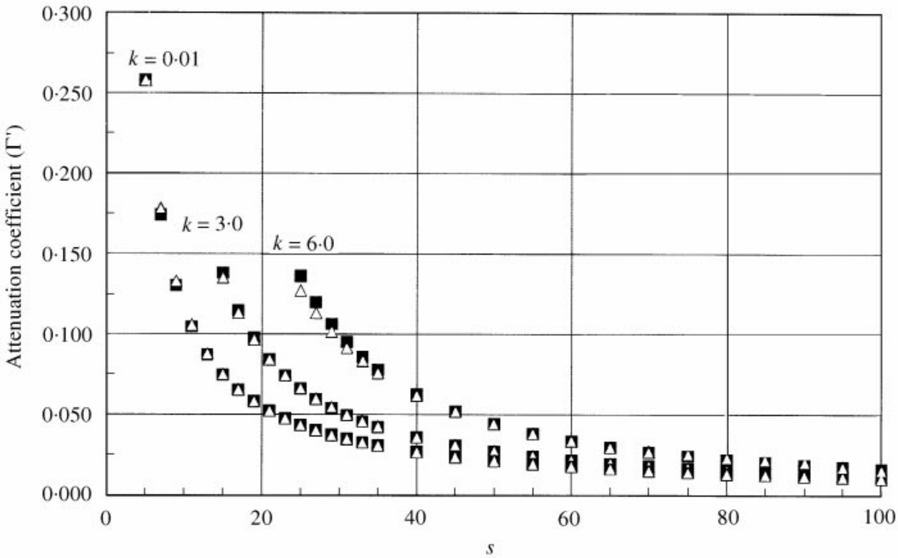

Figure 1. Comparison of polynomial fit (equation (9)) to exact results obtained from equation (1) for air: ■, Γ' exact; Δ, Γ'' fitted.

reference [1] and in reference [10] shows that the average error for the same 6464 points is 11·6%. The maximum errors for equation (9) and the Kirchhoff approximation over this range are 16·7 and 72·7% respectively.

Equation (10) showing the phase shift coefficient was fitted using a similar procedure. In this case the average and maximum errors over the 6464 points used to fit the equation are 0·04 and 1·6% respectively.

The purpose of equations (9) and (10) is to simplify computations. The most accurate way of solving for the propagation constant is to solve equation (1) using a numerical procedure such as the one outlined here. This numerical procedure, in our case, was implemented using the Mathematica software package since it has the capability of solving for the complex argument transcendental functions and because it was readily available to us. However, it may not be very practical to link this program to others, hence the need of simple equations like equation (9) and (10).

5. EXPERIMENTAL RESULTS

Work on noise from different air conditioning components [5–8] using refrigerant R134a lead us to this investigation of the visco-thermal sound attenuation in tubes. Kirchhoff's complete solution to the problem was obtained using the ideal gas equation of state. Refrigerants do not behave like ideal gases at normal superheated conditions. Tests reported here for sound attenuation in refrigerant R134a spanned a range of superheats between 12 and 28°C. These levels of superheat are not large enough that the refrigerant could be considered to

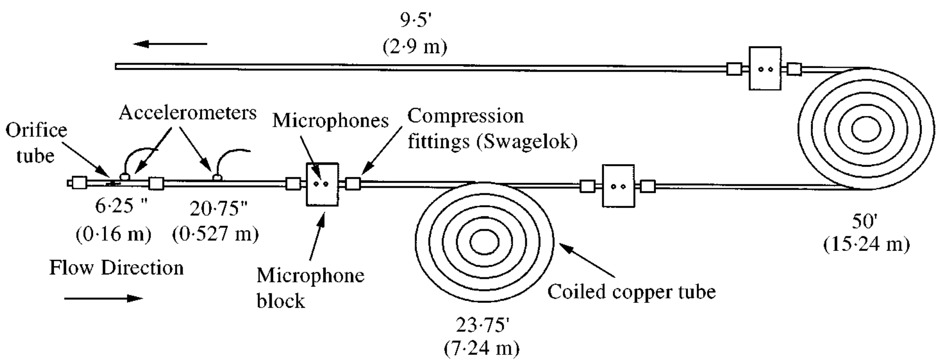

Figure 2. Experimental set-up used for studies of expansion device noise in refrigerant and where sound attenuation measurements were made. Tube dimensions 0·5 in OD (12·70 mm), 0·415 in ID (10·54 mm).

behave like an ideal gas. For this reason an error analysis was performed with the non-dimensional linearized version of the equation of state that is used to obtain the Kirchhoff's solution as shown in Tijdeman's paper. This equation is

$$P_a = T_a + \rho_a. \tag{11}$$

The above expression can be obtained when the total value of the pressure, temperature and density (the static and sinusoidally oscillating components) are introduced in the ideal gas equation of state. In the above equation P_a, T_a and ρ_a are the acoustic magnitudes non-dimensionalized by their respective static component. An acoustic perturbation was introduced and then the values of T_a and ρ_a were estimated using a real gas equation of state. The non-dimensionalized temperature and density acoustic perturbations obtained in this way were then compared to the initial acoustic perturbation introduced. The thermodynamic properties of refrigerant R134a were estimated using the Engineering Equation Solver (EES) software package version 5·002† that uses the Martin–Hou equation of state [11] based on thermodynamic information from McLinden *et al.* [12] for refrigerant R134a. The discrepancy found for R134a refrigerant at 578 kPa and 33°C is 8·2%. The error was not sensitive to the magnitude of the acoustic perturbation. For large superheats, say 150°C, the error is of the order of 2%. These error levels are approximately the same as those obtained for the speed of sound and density using ideal and real gas equations of state. In spite of this, sound attenuation measurements compare relatively well with theoretical predictions as can be seen in Figure 3.

Noise attenuation measurements were made in an experimental set-up built for noise studies of expansion devices (thermostatic expansion valves, orifice tubes, and capillary tubes) used in refrigeration and air conditioning systems [8]. Very long tube coils were used in the experimental set-up so that the visco-thermal

†Engineering Equation Solver (EES) Software Package version 5.002, F-chart Software, 4406 Fox Bluff Rd. Middleton WI 53562, U.S.A. http://www.fchart.com

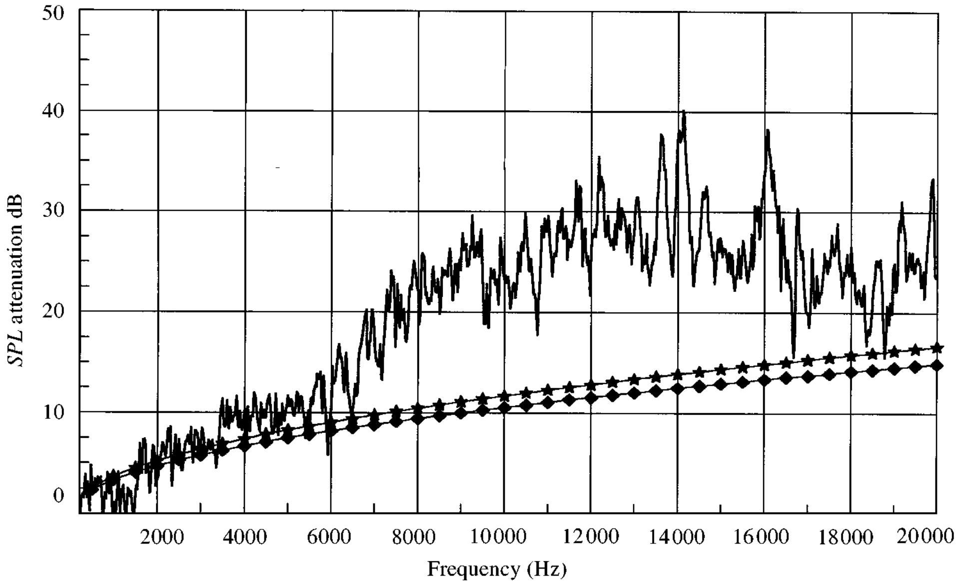

Figure 3. Sound attenuation measurements between microphone blocks separated by a 7·24 m coiled tube with 0·415 in (10·54 mm) ID. Fluid used is R134a refrigerant gas at $P = 5{\cdot}78$ bar, $T = 33°\mathrm{C}$, $c = 150{\cdot}8$ m/s. The first cut-off frequencies for this case are: $(1, 0) = 8385$ Hz, $(2, 0) = 13\,910$ Hz, $(0, 1) = 17\,450$ Hz, $(3, 0) = 19\,132$ Hz. ——, Experientally measured; ◆, estimated (wide tube approximation); ★, estimated (exact formulation).

attenuation through them would eliminate most of the acoustic reflections and practically provide for an anechoic termination. By doing this, a good characterization of expansion device noise could be achieved. Figure 2 shows the section of the set-up where the sound pressure and attenuation measurements were performed. A description of the complete experimental facility that shows in more detail the way in which the refrigerant is conditioned prior to enter the test section and after leaving the test section can be seen in references [8, 6].

The test section consists of an expansion device (Figure 2 shows an orifice tube), several microphone blocks (where the internal acoustic pressure was measured), and a set of very long tube coils. Refrigerant is fed into the orifice tube at a specified pressure and temperature or quality. The refrigerant, after passing through the test section, goes back to the system where it is condensed and reconditioned before returning to the test section.

The microphone blocks each hold two microphones. Even with the introduction of the extremely long tube sections low-frequency reflections are difficult to attenuate and the two-microphone technique is needed to account for low-frequency reflections (below 1 kHz) [8, 13].

Sound attenuation measurements were made by comparing readings of the noise at different positions in the test section. An HP3562A dynamic signal analyzer was used to process the signals from the dynamic pressure transducers that were placed in the microphone blocks shown in Figure 2. PCB model 105B02 dynamic pressure transducers were used. These transducers have a sensing tip of 0·1″ (2·5 mm) diameter and a resonance frequency of 250 kHz. The dynamic pressure transducers have a sensitivity of approx. 5·8 mV/kPa (40 mV/psi). These sensors are not very sensitive due to their small size. However, the expansion valve noise is typically 35–60 dB/$\sqrt{}$Hz greater than the instrumentation noise. The sensor deviation from linear behavior or linearity is reported by the manufacturer to be less than 2% of full scale. This translates to a maximum error in the measurement presented of the order of 0·4 dB/$\sqrt{}$Hz. These sensors where selected since they are capable of measuring the acoustic pressures on top of the relatively high static pressures seen in the low-pressure side of R134a refrigeration systems operating under typical conditions. Another important characteristic of these sensors is their size, which permit measurements inside of pipes of small diameter.

Figure 3 compares the sound attenuation measured when superheated R134a refrigerant flows through the test section to sound attenuation estimated using equation (1) and the wide tube approximation by Kirchhoff [1, 10].

Figure 3 shows good agreement between experimental measurements and theoretical results in most of the plane wave region. The noise source used during these measurements in an orifice tube (1·7 mm inside diameter, and 38·1 mm long). Typically, the sound pressure levels generated by this device for a number of different conditions of interest (when superheated refrigerant exits the orifice) are of the order of 100–130 dB/$\sqrt{}$Hz relative to 20E-6 Pa and the internal sound pressure spectrum remains basically constant at this level over the audible frequency range. Instrumentation noise varies almost linearly from around 75 dB/$\sqrt{}$Hz at lower frequencies down to about 60 dB/$\sqrt{}$Hz at 20 kHz. Since instrumentation noise approximately an order of magnitude greater at lower frequencies than at higher frequencies the signal to noise ratio at low frequencies will be smaller than at higher frequencies. Still the signal is close to two orders of magnitude larger at lower frequencies than the instrumentation noise.

The flow rate of refrigerant for the test presented in Figure 3 is 17·52 g/s (139·02 lb/h). This translates to an average flow velocity of 7·6 m/s or a Mach number of the flow of 0·05. This particular case represents a higher end flow velocity seen during our experiments. Measurements of sound attenuation in the test section under different conditions and at Mach numbers between 0·03 and 0·05 do not deviate from the results shown in Figure 3. However, mean flow can have an effect in the sound propagation in tubes as shown by some researchers [14–16]. Temperature gradients can also have an effect. In our experiments, the largest temperature gradients are of the order of 0·6°C/m with typical values less than this. In this work the temperature gradients effects are neglected. In cases of significant temperature gradients, Peat [17] presents a formulation which considers temperature gradient effects on sound propagation.

Figure 4 shows the experimental attenuation measurements of two data points taken. It is difficult to discriminate between the different curves. Figure 4 shows the repeatability of the experimentally measured sound attenuation.

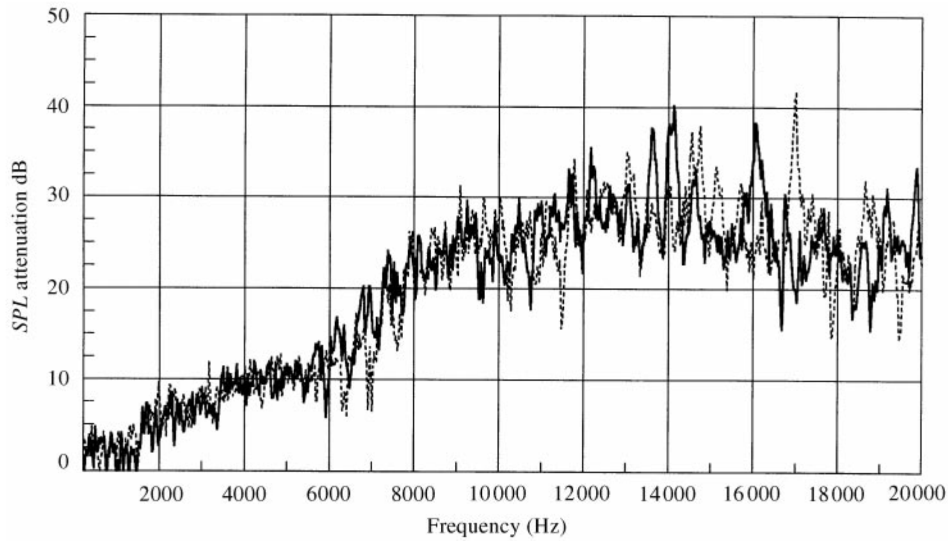

Figure 4. Sound attenuation measurements between microphone blocks separated by a 7·24 m coiled tube with 0·415 in (10·54 mm) ID. Fluid used is R134a refrigerant gas. ——, $P = 578$ kPa, $T = 33°$C, mass flow = 17·52 g/s; - - - -, $P = 567$ kPa, $T = 45°$C, mass flow = 13·18 g/s.

6. CONCLUSIONS

- The complex transcendental equation that describes the attenuation of sound waves in tubes due to visco-thermal effects was solved using a simple code (see footnote ‡) created using readily available software (see footnote †).
- The code was validated using previously published results for air and then used to create tables which are presented in Appendix A. The tables cover the range $5 < s < 5000$, $0{\cdot}01 < k < 6$, $0{\cdot}8 < \sigma < 1{\cdot}1$ and $1{\cdot}0 < \gamma < 1{\cdot}7$.
- Polynomial-type equations involving the relevant parameters were fitted in order to provide an alternative method calculation of the attenuation coefficient ($\mathrm{Re}\{\Gamma\}$) and phase shift coefficient ($\mathrm{Im}\{\Gamma\}$).
- Sound attenuation measurements of the sound propagating inside tubes for superheated R134a refrigerant agrees reasonably well with theory in the plane wave region. The superheat during our tests was not high enough that the refrigerant could be considered to behave as ideal gases. In spite of this the model predicts relatively well in the plane wave region.
- There are other dissipation mechanisms. However, for the case reported in this paper visco-thermal attenuation seems to be the dominant mechanism.

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APPENDIX A: VARIOUS TABLES

TABLE 1

Attenuation coefficient (Γ') for different values of γ and $\sigma = 0{\cdot}8$. First column shows values for air

s	k	Air $\gamma = 1{\cdot}4$ $\sigma = 0{\cdot}842$	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
5	0·01	0·258308	0·186382	0·205988	0·225016	0·243498	0·261462	0·278939	0·295953	0·312532
10	0·01	0·11584	0·081335	0·090584	0·099684	0·108638	0·117451	0·126126	0·134668	0·14308
20	0·01	0·054934	0·037933	0·042452	0·046934	0·051377	0·055783	0·060153	0·064487	0·0687845
35	0·01	0·030693	0·021034	0·023592	0·026137	0·028669	0·03119	0·033697	0·036193	0·0386767
50	0·01	0·021294	0·014547	0·016331	0·018108	0·019879	0·021644	0·023403	0·025158	0·026903
100	0·01	0·010537	0·007172	0·008059	0·008946	0·00983	0·010714	0·011595	0·012475	0·0133538
500	0·01	0·00209	0·001418	0·001595	0·001772	0·001949	0·002126	0·002302	0·002479	0·0026554
1000	0·01	0·001044	0·000708	0·000797	0·000885	0·000973	0·001062	0·00115	0·001238	0·0013268
2000	0·01	0·000522	0·000354	0·000398	0·000442	0·000486	0·000531	0·000575	0·000619	0·0006631
5000	0·01	0·000209	0·00015	0·000167	0·000185	0·000203	0·000221	0·000238	0·000258	0·0002735
5	1	0·325388	0·229015	0·254673	0·279992	0·304991	0·329691	0·354107	0·378258	0·402158
10	1	0·131909	0·091331	0·10209	0·112772	0·12338	0·133915	0·144381	0·154779	0·165112
20	1	0·058813	0·040331	0·04522	0·05009	0·054942	0·059774	0·064588	0·069384	0·0741621
35	1	0·031939	0·021802	0·024479	0·02715	0·029815	0·032474	0·035126	0·037772	0·0404124
50	1	0·0219	0·014921	0·016763	0·018601	0·020437	0·02227	0·024099	0·025926	0·0277494
100	1	0·010687	0·007264	0·008167	0·009068	0·009969	0·010869	0·011768	0·012667	0·0135643
500	1	0·002096	0·001422	0·001599	0·001777	0·001954	0·002132	0·002309	0·002486	0·0026638
1000	1	0·001045	0·000709	0·000798	0·000886	0·000975	0·001063	0·001152	0·00124	0·0013289
2000	1	0·000522	0·000354	0·000398	0·000442	0·000486	0·000531	0·000575	0·000619	0·0006637
5000	1	0·000209	0·000141	0·000159	0·000177	0·000194	0·000212	0·00023	0·000248	0·0002652
20	3	0·090417	0·059682	0·067611	0·075692	0·083926	0·092313	0·100854	0·10955	0·118402
35	3	0·042052	0·027992	0·031646	0·035349	0·0391	0·0429	0·04675	0·050649	0·0545986
50	3	0·026804	0·017925	0·020241	0·02258	0·024943	0·027329	0·029739	0·032172	0·0346293
100	3	0·011897	0·008006	0·009026	0·010051	0·011082	0·012118	0·01316	0·014208	0·0152616
500	3	0·002144	0·001451	0·001633	0·001816	0·001998	0·002181	0·002384	0·002547	0·002731

1000	3	0·001057	0·000716	0·000806	0·000896	0·000986	0·0001076	0·001166	0·001256	0·0013456
2000	3	0·000525	0·000356	0·0004	0·000445	0·000489	0·000534	0·000579	0·000623	0·0006679
5000	3	0·000209	0·000141	0·000159	0·000177	0·000194	0·000212	0·00023	0·000248	0·0002552
35	6	0·077729	0·049339	0·056496	0·063944	0·071695	0·079759	0·088152	0·096889	0·10599
50	6	0·04399	0·028258	0·032258	0·036393	0·040665	0·045078	0·049636	0·054342	0·0592009
100	6	0·016068	0·010539	0·011965	0·013421	0·014908	0·016426	0·017976	0·019557	0·0211707
500	6	0·002306	0·001551	0·001749	0·001948	0·002148	0·002349	0·002551	0·002755	0·0029591
1000	6	0·001098	0·000741	0·000835	0·000929	0·001023	0·001117	0·001212	0·001307	0·0014024
2000	6	0·000535	0·000362	0·000408	0·000453	0·000499	0·000544	0·00059	0·000636	0·000682
5000	6	0·000211	0·000143	0·000161	0·000179	0·000196	0·000214	0·000232	0·00025	0·0002682

Attenuation coefficient (Γ') for different values of γ and $\sigma = 0{\cdot}9$

s	k	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
5	0·01	0·186382	0·204081	0·221317	0·23811	0·254482	0·270454	0·286045	0·301273
10	0·01	0·081335	0·089648	0·097842	0·105919	0·113882	0·121734	0·129478	0·137116
20	0·01	0·037933	0·041972	0·045982	0·049961	0·05391	0·05783	0·061721	0·065584
35	0·01	0·021034	0·023315	0·025585	0·027846	0·030097	0·032338	0·034559	0·036791
50	0·01	0·014547	0·016136	0·01772	0·019299	0·020873	0·022442	0·024007	0·025567
100	0·01	0·007172	0·007962	0·00875	0·009538	0·010324	0·01111	0·011893	0·012676
500	0·01	0·001418	0·001576	0·001733	0·00189	0·002047	0·002204	0·002361	0·002518
1000	0·01	0·000708	0·000787	0·000865	0·000944	0·001022	0·001101	0·00118	0·001258
2000	0·01	0·000354	0·000393	0·000432	0·000472	0·000511	0·00055	0·00059	0·000629
5000	0·01	0·000141	0·000157	0·000173	0·000189	0·000204	0·00022	0·000236	0·000251
5	1	0·229015	0·252157	0·275039	0·297673	0·320071	0·342243	0·3642	0·38595
10	1	0·091331	0·100954	0·11052	0·120028	0·129481	0·13888	0·148228	0·157525
20	1	0·040331	0·044685	0·049024	0·053349	0·057659	0·061955	0·066237	0·070505
35	1	0·021802	0·024184	0·02658	0·028932	0·031299	0·033661	0·036018	0·03837
50	1	0·014921	0·016559	0·018194	0·019827	0·021458	0·023086	0·024712	0·026335
100	1	0·007264	0·008066	0·008868	0·009669	0·010469	0·011269	0·012068	0·012867
500	1	0·001422	0·00158	0·001737	0·001895	0·002053	0·002211	0·002368	0·002526
1000	1	0·000709	0·000788	0·000866	0·000944	0·001024	0·001103	0·001181	0·00126
2500	1	0·000283	0·000314	0·000346	0·000377	0·000409	0·00044	0·000472	0·000629
5000	1	0·000141	0·000157	0·000173	0·000189	0·000204	0·00022	0·000236	0·000251

TABLE 1

Continued

s	k	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
20	3	0·059682	0·066621	0·073682	0·080867	0·088175	0·095607	0·103162	0·110841
35	3	0·027992	0·031198	0·034442	0·037726	0·041049	0·044412	0·047815	0·051258
50	3	0·017925	0·019961	0·022016	0·02409	0·026183	0·028294	0·030425	0·032574
100	3	0·008006	0·008907	0·009811	0·010721	0·011634	0·012552	0·013475	0·014402
500	3	0·001451	0·001613	0·001775	0·001937	0·002099	0·002261	0·002424	0·002587
1000	3	0·000716	0·000796	0·000876	0·000956	0·001035	0·001115	0·001195	0·001275
2000	3	0·000356	0·000395	0·000435	0·000475	0·000514	0·000554	0·000593	0·000633
5000	3	0·000141	0·000157	0·000173	0·000189	0·000204	0·00022	0·000236	0·000251
35	6	0·049339	0·055497	0·061883	0·068504	0·075366	0·082477	0·089848	0·09749
50	6	0·028258	0·031712	0·035272	0·038941	0·04272	0·046612	0·05062	0·054745
100	6	0·010539	0·01178	0·013045	0·014334	0·015648	0·016986	0·01835	0·019739
500	6	0·001551	0·001726	0·001901	0·002078	0·002255	0·002433	0·002612	0·002792
1000	6	0·000741	0·000824	0·000907	0·000991	0·001074	0·001158	0·001242	0·001326
2000	6	0·000362	0·000402	0·000443	0·000483	0·000524	0·000564	0·000605	0·000646
5000	6	0·000143	0·000159	0·000175	0·00019	0·000206	0·000222	0·000238	0·000254

Attenuation coefficient (Γ') for different values of γ and $\sigma = 1{\cdot}0$

s	k	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
5	0·01	0·186382	0·202545	0·218326	0·233739	0·248802	0·26353	0·277936	0·292035
10	0·01	0·0813348	0·088883	0·096334	0·103689	0·11095	0·118119	0·125199	0·132191
20	0·01	0·037933	0·041585	0·045211	0·048813	0·052391	0·055946	0·059476	0·062983
35	0·01	0·0210339	0·023092	0·025141	0·027183	0·029216	0·031242	0·033259	0·035269
50	0·01	0·0145471	0·01598	0·017408	0·018833	0·020253	0·02167	0·023082	0·024491
100	0·01	0·00717169	0·007883	0·008594	0·009304	0·010012	0·01072	0·011427	0·012132
500	0·01	0·00141822	0·00156	0·001701	0·001843	0·001984	0·002126	0·002267	0·002409
1000	0·01	0·000708107	0·000779	0·00085	0·00092	0·000991	0·001062	0·001132	0·001203
2000	0·01	0·000353803	0·000389	0·000425	0·00046	0·000495	0·000531	0·000566	0·000601
5000	0·01	0·000141461	0·000156	0·00017	0·000184	0·000198	0·000212	0·000226	0·00024

5	1	0·229015	0·250071	0·270922	0·291575	0·312038	0·332317	0·352419	0·372349
10	1	0·0913308	0·100032	0·108687	0·117297	0·125864	0·134389	0·142873	0·151317
20	1	0·0403306	0·044255	0·048167	0·052068	0·055957	0·059835	0·063701	0·067557
35	1	0·021802	0·023947	0·026087	0·028224	0·030356	0·032485	0·03461	0·036731
50	1	0·0149206	0·016395	0·017868	0·019338	0·020807	0·022274	0·023738	0·025201
100	1	0·00726421	0·007986	0·008708	0·009429	0·010149	0·010869	0·011589	0·012308
500	1	0·00142189	0·001564	0·001706	0·001848	0·00199	0·002132	0·002274	0·002416
1000	1	0·000709025	0·00078	0·000851	0·000922	0·000992	0·001062	0·001134	0·001205
2000	1	0·000353803	0·000389	0·000425	0·00045	0·000495	0·000531	0·000566	0·000601
5000	1	0·000141461	0·000156	0·00017	0·000184	0·000198	0·000212	0·000226	0·00024
20	3	0·0596817	0·065844	0·072107	0·078472	0·084938	0·091506	0·098174	0·104945
35	3	0·0279921	0·030846	0·033731	0·03665	0·0396	0·042583	0·045599	0·048647
50	3	0·0179247	0·019741	0·021573	0·02342	0·025283	0·027161	0·029054	0·030963
100	3	0·00800647	0·008812	0·009622	0·010435	0·011252	0·012072	0·012896	0·013724
500	3	0·0014513	0·001597	0·001742	0·001888	0·002033	0·002179	0·002325	0·002471
1000	3	0·000716367	0·000788	0·00086	0·000932	0·001003	0·001075	0·001147	0·001219
2000	3	0·0003555867	0·000391	0·000427	0·000463	0·000498	0·000534	0·00057	0·000605
5000	3	0·000141461	0·000156	0·00017	0·000184	0·000199	0·000213	0·000227	0·000241
35	6	0·0493391	0·054732	0·060308	0·066072	0·072026	0·078178	0·084532	0·091096
50	6	0·028258	0·031292	0·034413	0·037621	0·040918	0·044306	0·047787	0·051362
100	6	0·0105387	0·011636	0·012753	0·01389	0·015046	0·016223	0·01742	0·018637
500	6	0·00155076	0·001707	0·001864	0·002022	0·002181	0·00234	0·0025	0·002861
1000	6	0·000741175	0·000816	0·00089	0·000965	0·00104	0·001115	0·001191	0·001266
2000	6	0·000362062	0·000398	0·000435	0·000471	0·000507	0·000544	0·00058	0·000617
5000	6	0·000142782	0·000157	0·000171	0·000186	0·0002	0·000214	0·000229	0·000243

Attenuation coefficient (Γ') for different values of γ and $\sigma = 1{\cdot}1$

s	k	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
5	0·01	0·186382	0·201276	0·215849	0·230112	0·244077	0·257757	0·271162	0·284303
10	0·01	0·081335	0·088246	0·095076	0·101826	0·108498	0·115092	0·121612	0·128057
20	0·01	0·037933	0·041264	0·044575	0·047866	0·051136	0·054386	0·057617	0·060828
35	0·01	0·021034	0·022908	0·024776	0·026637	0·028491	0·030339	0·03218	0·034015
50	0·01	0·014547	0·015851	0·017152	0·018449	0·019744	0·021035	0·022322	0·023607

TABLE 1

Continued

s	k	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
100	0·01	0·007172	0·007819	0·008466	0·009111	0·009756	0·0104	0·011044	0·011686
500	0·01	0·001418	0·001547	0·001676	0·001804	0·001933	0·002062	0·00219	0·002319
1000	0·01	0·000708	0·000772	0·000837	0·000901	0·000965	0·00103	0·001094	0·001158
2000	0·01	0·000354	0·000386	0·000418	0·00045	0·000482	0·000515	0·000547	0·000579
5000	0·01	0·000125	0·000138	0·000151	0·000163	0·000176	0·000189	0·000202	0·000215
5	1	0·229015	0·248329	0·267477	0·286464	0·305294	0·323972	0·342502	0·360887
10	1	0·091331	0·099268	0·107168	0·115033	0·122863	0·13066	0·138423	0·146154
20	1	0·040331	0·043902	0·047464	0·051016	0·054558	0·058092	0·061616	0·065131
35	1	0·021802	0·023752	0·025699	0·027643	0·029584	0·031521	0·033456	0·035387
50	1	0·014921	0·016261	0·017601	0·018938	0·020274	0·021608	0·022941	0·024272
100	1	0·007264	0·007921	0·008577	0·009232	0·009887	0·010542	0·011197	0·011851
500	1	0·001422	0·001551	0·00168	0·001809	0·001938	0·002067	0·002196	0·002325
1000	1	0·000709	0·000773	0·000838	0·000902	0·000967	0·001031	0·001095	0·00116
2000	1	0·000354	0·000386	0·000418	0·00045	0·000482	0·000515	0·000547	0·000579
5000	1	0·000141	0·000154	0·000167	0·00018	0·000193	0·000208	0·000219	0·000231
20	3	0·059682	0·065219	0·070842	0·07655	0·082343	0·088221	0·094183	0·10023
35	3	0·027992	0·030583	0·03316	0·035785	0·038436	0·041115	0·043821	0·046555
50	3	0·017925	0·019564	0·021216	0·02288	0·024558	0·026248	0·027952	0·029668
100	3	0·008006	0·008736	0·009468	0·010204	0·010942	0·011683	0·012428	0·013176
500	3	0·001451	0·001583	0·001715	0·001848	0·00198	0·002112	0·002245	0·002377
1000	3	0·000716	0·000781	0·000847	0·000912	0·000977	0·001042	0·001108	0·001173
2000	3	0·000356	0·000388	0·000421	0·000453	0·000485	0·000518	0·00055	0·000582
5000	3	0·000141	0·000154	0·000167	0·00018	0·000193	0·000206	0·000219	0·000232
35	6	0·049339	0·054129	0·05907	0·064163	0·069413	0·074822	0·080393	0·086132
50	6	0·028258	0·030961	0·033735	0·036581	0·039501	0·042496	0·045566	0·048713
100	6	0·010539	0·011522	0·012521	0·013537	0·014569	0·015618	0·016683	0·017766
500	6	0·001551	0·001692	0·001835	0·001978	0·002121	0·002265	0·00241	0·002555
1000	6	0·000741	0·000809	0·000876	0·000944	0·001012	0·00108	0·001149	0·001217
2000	6	0·000362	0·000395	0·000428	0·000461	0·000494	0·000527	0·00056	0·000593
5000	6	0·000143	0·000156	0·000169	0·000182	0·000195	0·000208	0·000221	0·000234

TABLE 2

Phase shift coefficient (Γ'') for different values of γ and $\sigma = 0{\cdot}8$. First column shows values for air

s	k	Air $\gamma = 1{\cdot}4$ $\sigma = 0{\cdot}842$	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
5	0·01	1·20121	1·13521	1·15274	1·17022	1·18764	1·20497	1·22222	1·23936	1·25639
10	0·01	1·10349	1·07004	1·07887	1·08768	1·09649	1·10529	1·11407	1·12283	1·13157
20	0·01	1·05204	1·03527	1·03969	1·04411	1·04852	1·05294	1·05735	1·06176	1·06616
35	0·01	1·02978	1·02019	1·02271	1·02524	1·02776	1·03029	1·03281	1·03533	1·03786
50	0·01	1·02085	1·01414	1·0159	1·01767	1·01944	1·02121	1·02297	1·02474	1·02651
100	0·01	1·01043	1·00707	1·00795	1·00884	1·00972	1·01061	1·01149	1·01237	1·01326
500	0·01	1·00209	1·00141	1·00159	1·00177	1·00194	1·00212	1·0023	1·00247	1·00265
1000	0·01	1·00104	1·00071	1·0008	1·00088	1·00097	1·00106	1·00115	1·00124	1·00133
2000	0·01	1·00052	1·00035	1·0004	1·00044	1·00049	1·00053	1·00057	1·00062	1·00056
5000	0·01	1·00021	1·00014	1·00016	1·00018	1·00019	1·00021	1·00023	1·00025	1·00027
5	1	1·18175	1·12372	1·13962	1·15534	1·17086	1·18617	1·20124	1·21608	1·23067
10	1	1·1017	1·06892	1·0776	1·08628	1·09494	1·10358	1·11221	1·12081	1·12939
20	1	1·05186	1·03515	1·03956	1·04396	1·04836	1·05276	1·05716	1·06156	1·06595
35	1	1·02974	1·02017	1·02269	1·02521	1·02773	1·03026	1·03278	1·0353	1·03782
50	1	1·02084	1·01413	1·0159	1·01766	1·01943	1·0212	1·02296	1·02473	1·0265
100	1	1·01043	1·00707	1·00795	1·00884	1·00972	1·0106	1·01149	1·01237	1·01326
500	1	1·00209	1·00141	1·00159	1·00177	1·00194	1·00212	1·0023	1·00247	1·00265
1000	1	1·00104	1·00071	1·0008	1·00088	1·00097	1·00106	1·00115	1·00124	1·00133
2000	1	1·00052	1·00035	1·0004	1·00044	1·00049	1·00053	1·00057	1·00062	1·00066
5000	1	1·00021	1·00014	1·00016	1·00018	1·00019	1·00021	1·00023	1·00025	1·00027
20	3	1·04799	1·03322	1·03723	1·0412	1·0451	1·04895	1·05273	1·05645	1·05009
35	3	1·02917	1·01987	1·02234	1·0248	1·02725	1·0297	1·03213	1·03456	1·03697
50	3	1·02066	1·01404	1·01579	1·01754	1·01928	1·02103	1·02277	1·0245	1·02624
100	3	1·01041	1·00706	1·00794	1·00882	1·0097	1·01059	1·01147	1·01235	1·01323
500	3	1·00209	1·00141	1·00159	1·00177	1·00194	1·00212	1·0023	1·00247	1·00265
1000	3	1·00104	1·00071	1·0008	1·00088	1·00097	1·000106	1·00115	1·00124	1·00133
2000	3	1·00052	1·00035	1·0004	1·00044	1·00049	1·00053	1·00057	1·00062	1·00066
5000	3	1·00021	1·00014	1·00016	1·00018	1·00019	1·00021	1·00023	1·00025	1·00027

TABLE 2

Continued

s	k	Air $\gamma = 1{\cdot}4$ $\sigma = 0{\cdot}842$	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
35	6	1·02268	1·01719	1·01891	1·02048	1·02189	1·02312	1·02416	1·02501	1·02565
50	6	1·01878	1·01326	1·0148	1·01628	1·01772	1·01911	1·02044	1·02171	1·02292
100	6	1·01022	1·00698	1·00784	1·0087	1·00955	1·01039	1·01123	1·01207	1·01289
500	6	1·00208	1·00141	1·00159	1·00177	1·00194	1·00212	1·0023	1·00247	1·00265
1000	6	1·00104	1·00071	1·0008	1·00088	1·00097	1·00106	1·00115	1·00124	1·00133
2000	6	1·00052	1·00035	1·0004	1·00044	1·00049	1·00053	1·00057	1·00062	1·00066
5000	6	1·00021	1·00014	1·00016	1·00018	1·00019	1·00021	1·00023	1·00025	1·00027

Phase shift coefficient (Γ'') for different values of γ and $\sigma = 0{\cdot}9$

s	k	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
5	0·01	1·13521	1·15066	1·1681	1·18149	1·19683	1·21211	1·22732	1·24245
10	0·01	1·07004	1·07788	1·08571	1·09353	1·10135	1·10915	1·11695	1·12473
20	0·01	1·03527	1·0392	1·04313	1·04705	1·05097	1·05489	1·05881	1·06273
35	0·01	1·02019	1·02243	1·02468	1·02692	1·02916	1·03141	1·03365	1·03589
50	0·01	1·01414	1·01571	1·01728	1·01885	1·02042	1·02199	1·02356	1·02513
100	0·01	1·00707	1·00786	1·00864	1·00943	1·01021	1·011	1·01178	1·01257
500	0·01	1·00141	1·00157	1·00173	1·00189	1·00204	1·0022	1·00236	1·00251
1000	0·01	1·00071	1·00079	1·00086	1·00094	1·00102	1·0011	1·00118	1·00126
2000	0·01	1·00035	1·00039	1·00043	1·00047	1·00051	1·00055	1·00059	1·00063
5000	0·01	1·00014	1·00016	1·00017	1·00019	1·0002	1·00022	1·00024	1·00025
5	1	1·12372	1·13724	1·15059	1·16377	1·17677	1·18956	1·20216	1·21454
10	1	1·06892	1·07657	1·08422	1·09185	1·09947	1·10707	1·11465	1·12221
20	1	1·03515	1·03906	1·04297	1·04687	1·05078	1·05488	1·05858	1·06248
35	1	1·02017	1·02241	1·02465	1·02689	1·02913	1·03137	1·03361	1·03585
50	1	1·01413	1·0157	1·01727	1·01864	1·02041	1·02198	1·02355	1·02512
100	1	1·00707	1·00786	1·00864	1·00943	1·01021	1·011	1·01178	1·01257
500	1	1·00141	1·00157	1·00173	1·00189	1·00204	1·0022	1·00238	1·00251
1000	1	1·00071	1·00079	1·00086	1·00094	1·00102	1·0011	1·00118	1·00126

2000	1	1·00028	1·00031	1·00035	1·00038	1·00041	1·00044	1·00047	1·00063
5000	1	1·00014	1·00016	1·00017	1·00019	1·0002	1·00022	1·00024	1·00025
20	3	1·03322	1·03671	1·04016	1·04355	1·04688	1·05016	1·05338	1·05653
35	3	1·01987	1·02205	1·02423	1·02639	1·02855	1·0307	1·03285	1·03498
50	3	1·01404	1·01559	1·01714	1·01869	1·02023	1·02177	1·02331	1·02485
100	3	1·00706	1·00784	1·00863	1·00941	1·01019	1·01097	1·01176	1·01254
500	3	1·00141	1·00157	1·00173	1·00189	1·00204	1·0022	1·00236	1·00251
1000	3	1·00071	1·00079	1·00086	1·00094	1·00102	1·00011	1·00118	1·00126
2000	3	1·00035	1·00039	1·00043	1·00047	1·00051	1·00055	1·00059	1·00083
5000	3	1·00014	1·00016	1·00017	1·00019	1·0002	1·00022	1·00024	1·00025
35	6	1·01719	1·01865	1·01997	1·02115	1·02219	1·02306	1·02377	1·0243
50	6	1·01326	1·0146	1·01591	1·01717	1·01839	1·01956	1·02068	1·02175
100	6	1·00698	1·00775	1·0085	1·00926	1·01001	1·01075	1·01149	1·01223
500	6	1·00141	1·00157	1·00173	1·00188	1·00204	1·0022	1·00236	1·00251
1000	6	1·00071	1·00079	1·00086	1·00094	1·00102	1·0011	1·00118	1·00126
2000	6	1·00035	1·00039	1·00043	1·00047	1·00051	1·00055	1·00059	1·00063
5000	6	1·00014	1·00016	1·00017	1·00019	1·0002	1·00022	1·00024	1·00025

Phase shift coefficient (Γ'') for different values of γ and $\sigma = 1{\cdot}0$

s	k	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
5	0·01	1·13521	1·14908	1·16293	1·17677	1·19056	1·20432	1·21802	1·23167
10	0·01	1·07004	1·07709	1·08413	1·09117	1·0982	1·10522	1·11224	1·11925
20	0·01	1·03527	1·03881	1·04234	1·04587	1·0494	1·05293	1·05646	1·05999
35	0·01	1·02019	1·02221	1·02423	1·02625	1·02827	1·03029	1·0323	1·03432
50	0·01	1·01414	1·01555	1·01697	1·01838	1·01979	1·02121	1·02262	1·02403
100	0·01	1·00707	1·00778	1·00848	1·00919	1·0099	1·01061	1·01131	1·01202
500	0·01	1·00141	1·00156	1·0017	1·00184	1·00198	1·00212	1·00226	1·0024
1000	0·01	1·00071	1·00078	1·00085	1·00092	1·00099	1·00106	1·00113	1·0012
2000	0·01	1·00035	1·00039	1·00042	1·00046	1·00049	1·00053	1·00057	1·0006
5000	0·01	1·00014	1·00016	1·00017	1·00018	1·0002	1·00021	1·00023	1·00024
5	1	1·12372	1·1355	1·14714	1·15862	1·16994	1·18108	1·19205	1·20283
10	1	1·06892	1·07577	1·0826	1·08943	1·09624	1·10304	1·10981	1·11658
20	1	1·03515	1·03866	1·04217	1·04568	1·04919	1·0527	1·0562	1·05971
35	1	1·02017	1·02218	1·0242	1·02621	1·02823	1·03025	1·03226	1·03428
50	1	1·01413	1·01554	1·01696	1·01837	1·01978	1·02119	1·02261	1·02402

TABLE 2

Continued

s	k	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
100	1	1·00707	1·00778	1·00848	1·00919	1·0099	1·0106	1·01131	1·01202
500	1	1·00141	1·00156	1·0017	1·00184	1·00198	1·00212	1·00228	1·0024
1000	1	1·00071	1·00078	1·00085	1·00092	1·00099	1·00106	1·00113	1·0012
2000	1	1·00035	1·00039	1·00042	1·00046	1·00049	1·00053	1·00057	1·0006
5000	1	1·00014	1·00016	1·00017	1·00018	1·0002	1·00021	1·00023	1·00024
20	3	1·03322	1·03631	1·03936	1·04237	1·04532	1·04822	1·05106	1·05385
35	3	1·01987	1·02183	1·02378	1·02572	1·02765	1·02958	1·0315	1·03341
50	3	1·01404	1·01543	1·01682	1·01821	1·0196	1·02099	1·02237	1·02375
100	3	1·00706	1·00776	1·00847	1·00917	1·00988	1·01058	1·01129	1·01199
500	3	1·00141	1·00156	1·0017	1·00184	1·00198	1·00212	1·00226	1·0024
1000	3	1·00071	1·00078	1·00085	1·00092	1·00099	1·00106	1·00113	1·0012
2000	3	1·00035	1·00039	1·00042	1·00046	1·00049	1·00053	1·00057	1·0006
5000	3	1·00014	1·00016	1·00017	1·00018	1·0002	1·00021	1·00023	1·00024
35	6	1·01719	1·01846	1·01961	1·02064	1·02154	1·02231	1·02294	1·02341
50	6	1·01326	1·01446	1·01562	1·01674	1·01783	1·01888	1·01989	1·02085
100	6	1·00698	1·00767	1·00835	1·00903	1·0097	1·01037	1·01104	1·0117
500	6	1·00141	1·00155	1·0017	1·00184	1·00198	1·00212	1·00226	1·0024
1000	6	1·00071	1·00078	1·00085	1·00092	1·00099	1·00106	1·00113	1·0012
2000	6	1·00035	1·00039	1·00042	1·00046	1·00049	1·00053	1·00057	1·0006
5000	6	1·00014	1·00016	1·00017	1·00018	1·0002	1·00021	1·00023	1·00024
Phase shift coefficient (Γ'') for different values of γ and $\sigma = 1{\cdot}1$									
s	k	$\gamma = 1{\cdot}0$	$\gamma = 1{\cdot}1$	$\gamma = 1{\cdot}2$	$\gamma = 1{\cdot}3$	$\gamma = 1{\cdot}4$	$\gamma = 1{\cdot}5$	$\gamma = 1{\cdot}6$	$\gamma = 1{\cdot}7$
5	0·01	1·13521	1·14779	1·16037	1·17293	1·18547	1·19798	1·21046	1·22289
10	0·01	1·07004	1·07644	1·08284	1·08923	1·09562	1·10201	1·10839	1·11477
20	0·01	1·03527	1·03849	1·0417	1·04491	1·04812	1·05132	1·05453	1·05774
35	0·01	1·02019	1·02202	1·02386	1·0257	1·02753	1·02937	1·0312	1·03304

50	0·01	1·01414	1·01542	1·01671	1·01799	1·01928	1·02056	1·02185	1·02313
100	0·01	1·00707	1·00771	1·00836	1·009	1·00964	1·01028	1·01093	1·01157
500	0·01	1·00141	1·00154	1·00167	1·0018	1·00193	1·00206	1·00219	1·00231
1000	0·01	1·00071	1·00077	1·00084	1·0009	1·00096	1·00103	1·00109	1·00116
2000	0·01	1·00035	1·00039	1·00042	1·00045	1·00048	1·00051	1·00055	1·00058
5000	0·01	1·00014	1·00015	1·00017	1·00018	1·00019	1·00021	1·00022	1·00023
5	1	1·12372	1·13417	1·14448	1·15465	1·16468	1·17455	1·18426	1·19381
10	1	1·06892	1·07511	1·0813	1·08747	1·09363	1·09978	1·10591	1·11203
20	1	1·03515	1·03834	1·04153	1·04472	1·0479	1·05109	1·05427	1·05745
35	1	1·02017	1·022	1·02383	1·02566	1·02749	1·02933	1·03116	1·03299
50	1	1·01413	1·01541	1·0167	1·01798	1·01927	1·02055	1·02183	1·02312
100	1	1·00707	1·00771	1·00835	1·009	1·00964	1·01028	1·01093	1·01157
500	1	1·00141	1·00154	1·00167	1·0018	1·00193	1·00206	1·00219	1·00231
1000	1	1·00071	1·00077	1·00084	1·0009	1·00096	1·00103	1·00109	1·00116
2000	1	1·00035	1·00039	1·00042	1·00045	1·00048	1·00051	1·00055	1·00058
5000	1	1·00014	1·00015	1·00017	1·00018	1·00019	1·00021	1·00022	1·00023
20	3	1·03322	1·036	1·03874	1·04144	1·04409	1·04669	1·04925	1·05175
35	3	1·01987	1·02165	1·02341	1·02517	1·02692	1·02867	1·03041	1·03214
50	3	1·01404	1·0153	1·01657	1·01783	1·01909	1·02035	1·0216	1·02286
100	3	1·00706	1·0077	1·00834	1·00898	1·00962	1·01026	1·0109	1·01154
500	3	1·00141	1·00154	1·00187	1·0018	1·00193	1·00206	1·00219	1·00231
1000	3	1·00071	1·00077	1·00084	1·0009	1·00096	1·00103	1·00109	1·00116
2000	3	1·00035	1·00039	1·00042	1·00045	1·00048	1·00051	1·00055	1·00058
5000	3	1·00014	1·00015	1·00017	1·00018	1·00019	1·00021	1·00022	1·00023
35	6	1·01719	1·01831	1·01934	1·02026	1·02107	1·02176	1·02233	1·02277
50	6	1·01326	1·01434	1·01539	1·01641	1·01739	1·01835	1·01927	1·02015
100	6	1·00698	1·0076	1·00822	1·00884	1·00945	1·01006	1·01067	1·01127
500	6	1·00141	1·00154	1·00167	1·0018	1·00193	1·00206	1·00218	1·00231
1000	6	1·00071	1·00077	1·00084	1·0009	1·00096	1·00103	1·00109	1·00116
2000	6	1·00035	1·00039	1·00042	1·00045	1·00048	1·00051	1·00055	1·00058
5000	6	1·00014	1·00015	1·00017	1·00018	1·00019	1·00021	1·00022	1·00023

APPENDIX B: NOMENCLATURE

A, B	constants associated with equation (4)
$a1, a2, a3$	auxiliary variables
c	speed of sound
C_p	constant pressure specific heat
C_v	constant volume specific heat
i^2	-1
$\mathrm{J}_m(\ )$	Bessel function of the first kind of order m
k	reduced frequency
n	iteration number in equation (5), auxiliary variable in equation (7)
P	pressure
P_{ac}	acoustic pressure
P_a	non-dimensionalized acoustic pressure
R	internal tube radius
s	Stokes number or shear wave number
T	temperature
T_a	non-dimensionalized acoustic temperature
Z	propagation constant squared
γ	ratio of specific heats
Γ	propagation coefficient
Γ'	attenuation coefficient ($\mathrm{Re}\{\Gamma\}$)
Γ''	phase shift coefficient ($\mathrm{Im}\{\Gamma\}$)
ξ	$\omega \cdot x/c$
σ	square root of Prandtl number
λ	thermal conductivity
ρ	mean density
ρ_a	non-dimensionalized acoustic density
μ	absolute fluid viscosity
ω	frequency (rad/s)
χ_1, χ_2	auxiliary variables